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Procedia Computer Science 64 (2015) 265 – 273

Procedia Computer Science

Conference on ENTERprise Information Systems / International Conference on Project MANagement / Conference on Health and Social Care Information Systems and Technologies, CENTERIS / ProjMAN / HCist 2015 October 7-9, 2015

Redundancy resource allocation for reliable project scheduling: A game-theoretical approach

M. Rohaninejad[a]*, R. Tavakkoli-Moghaddam[b], B. Vahedi-Nouri[c]

[a]*MAPSA Company, Project Management R&D Department, Tehran, Iran*
[b]*School of Industrial Engineering and Engineering Optimization Research Group, College of Engineering, University of Tehran, Tehran, Iran*
[c]*Department of Industrial Engineering, College of Engineering, Bu-Ali Sina University, Hamedan, Iran*

Abstract

The redundancy allocation problem is among the most interesting and difficult problems in the system reliability design. In this paper this concept is considered to enhance reliability in projects network scheduling with stochastic activity duration. In order to determine the optimal manner of redundancy allocation, a new mathematical model is developed. Then, by simulating the problem in the form of a game-theoretical pattern, it is shown that the Nash-equilibrium points of the problem are very close to optimal solutions of original model. Therefore, an algorithmic approach is developed for the calculation of Nash equilibria. Finally, several computational experiments are executed and their results are analysed. The comparison of equilibrium outcomes with the optimal policy justifies the efficiency of Nash equilibria for increasing the projects network reliability.

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Keywords: Project scheduling, Reliability, Redundancy resource allocation, Game theory.

1. Introduction

The vast majority of research in the area of project scheduling presume that the factors regarding the project scheduling problem are deterministic but in reality, project activities are subject to considerable uncertainty. Main

* Corresponding author. Tel.: +989126040581
E-mail address: rohaninejad@mapsaeng.com

Peer-review under responsibility of SciKA - Association for Promotion and Dissemination of Scientific Knowledge
doi:10.1016/j.procs.2015.08.489

sources of uncertainty can be: duration of activities, resource consumption, resource availability, stochastic task insertion failures of equipment, customer's acceptance or refusal at different phases of a project etc.

In this paper we considered a reliability optimization in project scheduling with stochastic activity time. This problem involves finding a suitable allocation mode of the resource to activities network possibly under system constraints. In other word, the problem is to select redundancy-levels and activity time distribution mode to maximize project scheduling reliability, given resource constraint. The redundancy allocation problem is one of the main branches of reliability optimization problems. The maximize system reliability by redundancy resource allocation, activity time distribution selection and considering resource capacity becomes a combinatorial optimization problem. In the formulation of a maximize reliability of project scheduling problem, for each project activity multiple time distribution choices (assuming different activity duration distributions need different level of resource) are used. According to figure 1 multiple activity duration distributions are available for each activity that each distribution supports a level of reliability (probability of completion time be less than total float). Thus, accessing to higher levels of reliability is possible by injecting more redundant resource. By resource redundancy, resources able to process a given activity were multiplied, which provided a kind of flexibility and made it possible to put identical resources on standby. The objective of our probabilistic problem is to build a schedule that has the greatest probability of attaining a optimal performance. The other conditions of the model are as follows:

- The amount of a redundant resource is capacitated.
- Each activity distribution related to project activities requires a predefined level of resource.
- The durations of activities are as a Beta distribution.
- The activities are planned in as soon as possible mode.
- There is no constraint for any activity on start and finish time.
- The redundant resources such as cash are applicable for all activities.

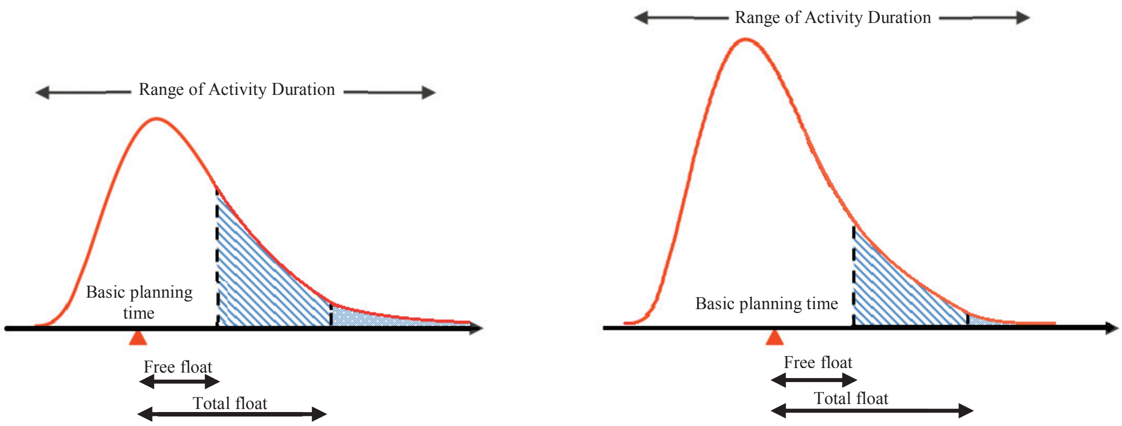

Fig. 1. Two duration distribution modes for an activity with different levels of reliability.

2. Literature Review

As mentioned previously , the scheduling problems are classified as both deterministic scheduling and scheduling with uncertainty. Deterministic scheduling has been studied extensively (i.e. [1-4]). In stochastic model of scheduling to tackle these uncertainties five approaches of scheduling are classified by Herroelen and Leus [5]: reactive approaches (i.e. Alagoz and Azizoglu [6]), stochastic approaches (i.e. Fang *et al.* [7]), fuzzy approaches (i.e. Wang [8]), proactive approaches (i.e. Lamas Vilches and Demeulemeester [9]) and approaches based on the sensitivity analysis (i.e. Penz *et al.* [10]). A comprehensive review of these approaches is provided by Herroelen and Leus [5], Brčić *et al.* [11] and Chaari *et al.* [12].

Proactive (also known as robust) scheduling approaches take uncertainty into account when designing off-line schedules. The scheduling takes future disruptions into consideration during the generation of the initial schedule.

This kind of approach tries to anticipate uncertainty while developing flexibility, in order to produce a schedule, or a family of schedules, that is relatively insensitive to uncertainty [13].

The literature regarding uncertainty of resources in the resource-constrained project scheduling problem is very limited. Lambrechts *et al.* [14] considered unforeseen breakdowns as a resource uncertainty. Their objective was to build a robust schedule that meets the project deadline and minimizes the schedule instability cost, defined as the expected weighted sum of the absolute deviations between the planned and the actually realized activity starting times during project execution. Lambrechts *et al.* [15] presented a tabu search procedure for this problem to achieve robust predictive project schedules. Lambrechts *et al.* [16] analytically determined the impact of unexpected resource breakdowns on activity durations. Furthermore, they developed an approach for inserting explicit idle time into the project schedule in order to protect it as well as possible from disruptions caused by resource unavailabilities. Ji and Yao [17] presented an uncertain project scheduling problem, of which both the duration times and the resources allocation times were uncertain variables. They developed an uncertain programming model with multiple objectives, whose first objective is to minimize the total cost, and second objective is to minimize the overtime. Then a Genetic algorithm was employed to solve the proposed model. Fu *et al.* [18] focused problems where the durations of activities are stochastic and resources can have unforeseen breakdowns. Given a level of allowable risk, α, their mechanisms aimed to compute the minimum robust makespan execution strategy. Robust makespan for an execution strategy is any makespan value that has a risk less than α. Firstly; they provided an analytical evaluation of resource breakdowns and repairs on executions of activities. Then incorporated such information into a local search framework and generated execution strategies that can absorb resource and durational uncertainties. Finally, to improve robustness of resulting strategies, they proposed resource breakdown aware chaining procedure with three different metrics. This chaining procedure computes resource allocations by predicting the effect of breakdowns on robustness of generated strategies. The redundancy allocation problem is a widely investigated topic in the field of reliability optimization. They used optimization methods into integer programming, dynamic programming, linear programming, generalized Lagrangian functions, and heuristic approaches. The most of them divided optimal system reliability models with redundancy into networks with patterns includes series, parallel, series-parallel, parallel-series or complex pattern. According to our best knowledge there is no literature for redundancy allocation problem for optimizing project network reliability.

3. Notation and model formulation

In this section, a new MIP model is presented. The parameters of the model that is called model 1 are as follows.

D	The due date of project
C	Total capacity of redundant resource.
N	The total number of project paths. ($n = 1, \dots, N$)
S	The total number of project activities. ($s = 1, \dots, S$)
E_n	The set of activities inside the *n*-th path.
j_s	The total number of time distribution for *s*-th activity of project. ($j = 1, \dots, j_s$)
$b_{j,s}$	The mean of the activity s duration when this activity is performed with the duration distribution j.
$a_{j,s}$	The variance of the activity s duration when it is performed with the duration distribution j.
f_s	The total float of activity s in basic scheduling.
$R_{j,s}$	The redundant resource needed to perform activity s when it is performed with duration distribution j.
$T_{n,n'}$	The set of common activities between *n*-th path and n'-th path.
A_n	The event in which the *n*-th path has been delayed.

The variables of the model are given below.

$y_{j,s}$	A binary variable that gets the value of 1 if the activity s is performed with the duration distribution j; otherwise, it is equal to 0.
μ_n	The mean of path n duration
δ_n	The standard deviation of path n duration

$p(x_s + x_{s'})$	The probability that the sum of duration for s and s'activities cause the delay in project.
$p(A_n)$	The delay probability of the *n*-th path $\left(p(A_n) = p\left(\sum_{s=1\vert s\in E_n}^{S} x_s\right)\right)$
$p(A_n A_{n'})$	The delay probability of the *n*-th path and n'-th path together.

The relation (1) indicates the problem's objective function, which minimizes the probability of project delay. The probability of project delay has high computational complexity due to the dependence of random variables related to the completion time of some project paths. In other words, the probabilities of delay for some project paths that have common activities are dependent. So, we tried to present an accurate approximation of this probability. However, this approximation is not appropriate for projects with low reliability paths. Nevertheless, the accuracy of this approximation will increase by the growth of the reliability of project paths. So that, the difference between them is very small and negligible if project paths have reliability greater than 0.9. In this paper, we used this approximation because the concept of reliable project schedule is not compatible with the low reliability values

$$\text{Min} \sum_{n=1}^{N} p(A_n) - \sum_{n=1}^{N} \sum_{n'=1|n'\neq n}^{N} p(A_n, A_{n'}) + \ \dots \ + (-1)^N p(A_n, A_{n'}, \dots, A_N) \cong \sum_{n=1}^{N} p(A_n) - \sum_{n=1}^{N} \sum_{n'=1|n'\neq n}^{N} p(A_n, A_{n'}) \qquad (1)$$

The constraint (2) specifies duration distribution mode for each activity.

$$\sum_{j=1}^{j_s} y_{j,s} = 1 \qquad s = 1,..,S \qquad (2)$$

The constraints (3) and (4) set the mean and standard deviation of project paths duration according to the mean and standard deviation of their components (activities) duration. These relations define base on the *central limit* theorem. According to *central limit* theorem if $x_1, x_2, \dots, x_N$ be a set of *N* independent random variables and each x_i has an arbitrary probability distribution $P(x_1, x_2, \dots, x_N)$ with mean μ_i and a finite variance δ_i^2. Then the $\sum_{i=1}^{N} x_i$ is a random variety that has a limiting cumulative distribution function which approaches a normal distribution with mean $\mu = \sum_{i=1}^{N} \mu_i$ and variance $\delta = \sqrt{\sum_{i=1}^{N} \delta_i^2}$. Kallenberg [19] gives a proof of the central limit theorem.

$$\mu_n \geq \sum_{s=1|s\in E_n}^{S} \sum_{j=1}^{j_s} y_{j,s}\, b_{j,s} \qquad n = 1,..,N \qquad (3)$$

$$\delta_n \geq \left(\sum_{s=1|s\in E_n}^{S} \sum_{j=1}^{j_s} y_{j,s}\, a_{j,s} \right)^{1/2} \qquad n = 1,..,N \qquad (4)$$

The constraint (5) calculates the delay's probability of project paths base on the *central limit* theorem.

$$p(A_n) \geq \int_D^{\infty} \left(\frac{1}{\sqrt{2\pi}\delta_n} e^{-(x-\mu_n)^2 / 2\delta_n^2} \right) dx \qquad n = 1,..,N \qquad (5)$$

The constraint (6) calculates the probability of simultaneous delay for paths n and n' when the random variables related to their completion time are dependent. We present a simple example to prove this relation. According to figure 2 there are three common activities (A, B and F) for path 1 includes $A \to B \to C \to E \to F$ and path 2 includes $A \to B \to D \to F$. So, the process of achieving relation that provide the probability of simultaneous delay for paths 1 and 2 is as follows:

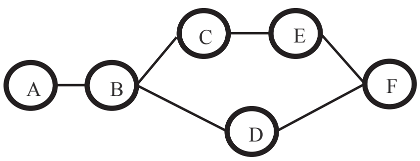

Fig. 2. The two path of a network with common activities.

$p(A_1) = p(A_1|A_2)\,p(A_2) + p(A_1|A_2')\,p(A_2') = p(A_1A_2) + \; p(A_1|A_2')\,p(A_2') \quad =>$
$p(A_1A_2) = p(A_1) - p(A_1|A_2')\,p(A_2')$

It can be found that the total duration of activities A, B and F is less than sum of their total float if path 2 is not delayed. In this condition, the path 1 can be delayed only if the total duration of activities C and E be greater than the the sum of their total float. Therefore, it can be concluded that $p(A_1|A_2') = p(x_C + x_E)$.

$p(A_1A_2) = p(A_1) - p(x_C + x_E)\,\big(1 - p(A_2)\big)$

The constraint (6) is the extension of above relation. Also, the constraint (7) calculates the probability of simultaneous delay for paths n and n' when the random variables related to their completion time are independent.

$$p(A_nA_{n'}) \le p(A_n) - p\left(\sum\nolimits_{s=1|s\notin T_{n,n'}\&s\in E_n}^{S} x_s\right)\big(1 - p(A_{n'})\big) \qquad n, n' = 1,..,N \;\&\; T_{n,n'} \neq \emptyset \qquad (6)$$

$$p(A_nA_{n'}) \le p(A_n)p(A_{n'}) \qquad n, n' = 1,..,N \;\&\; T_{n,n'} = \emptyset \qquad (7)$$

The constraint (8) calculates the probability that the sum of duration related to a number of different activities cause delay in project. In this relation, we assume for each paths n and $n'$the number of activities (|s|) when $s \notin T_{n,n'} \& \; s \in E_n$ and $T_{n,n'} \neq \emptyset$ is large enough that normal distribution is a suitable distribution for $(\sum_s x_s)$.Otherwise, we disregard one of the paths n and n′ due to high similarity of them (a lot common activities).

$$p\left(\sum_{s\notin T_{n,n'}\&\,s\in E_n} x_s\right) \ge \int_{\sum_s f_s}^{\infty} \frac{1}{\sqrt{2\pi}\left(\sum_s \sum_{j=1}^{j_s} y_{j,s}a_{j,s}\right)^{1/2}} e^{-\left(x-\sum_s\sum_{j=1}^{j_s} y_{j,s}b_{j,s}\right)^2 / 2\sum_s\sum_{j=1}^{j_s} y_{j,s}a_{j,s}} dx \qquad \begin{matrix} n, n' = 1,..,N \;\& \\ T_{n,n'} \neq \emptyset \end{matrix} \qquad (8)$$

The constraint (9) guarantees that the capacity limitation of redundant resource is abided by. In this relation, it is assumed for each activity s the parameter $j = 1$ is belong to the basic duration distribution selected in the preliminary planning. So, the value of $R_{1,s}$ is equal to zero.

$$\sum_{s=1}^{S}\sum_{j=1}^{j_s} R_{j,s}.\,y_{j,s} \le C \qquad (9)$$

3.1. Mathematical model (Model 2):

In this section we present a substitute model that is simpler than model 1. Numerical experiments show that the results of this model are very close to original model. This model that is called model 2 is as follows:

$$Max\left(Min_{n=1}^{N}\left(\int_{-\infty}^{D}\frac{1}{\sqrt{2\pi}\delta_n}e^{-(x-\mu_n)^2 / 2\delta_n{}^2}dx\right)\right) \qquad (10)$$

s.t.
Constraints 2, 3, 4 and 9.

The considered concept for model 2 is based on strengthening the weakest member of the set (project path with the lowest reliability). The project reliability (the probability of encountering no delay) is heavily influenced by weakest member. In other words, in order to avoid delay, all of the project paths must be finished before the project due date. As a result, existence of only one path with low reliability even in situations where other paths have high reliability can hinder the project in to terminate in time. The delays of low reliability path alone and independent of other paths will cause the delay in project.

4. The game- theoretical approach

In this section, we define problem in the form of a game-theoretical pattern and find equilibria for the game. Therefore, an algorithmic approach is developed for calculation of Nash equilibria. For defining Nash Equilibrium (NE), we consider a game with N players where S_i is the strategy set for player i. Let x_i be a strategy profile of player i and x_{-i} be a strategy profile of all players except player i. We show the set of strategies chosen by all players with $x = \{x_1, x_2, \dots, x_N\}$ and payoff obtained by player i with $f_i(x)$. Accordingly, a strategy profile x^* is Nash equilibrium if:

$$\forall\, i, x_i \in S_i \,:\; f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*) \tag{11}$$

In other words, In Nash point each player obtains the maximum of his/her profit against the received utility of other competitors. For our problem, the set of project paths make the game players (path n is equivalent of player n). The payoff for player n is the reliability of path n and each player has a tendency to increase its reliability by attracting more resource. Since the set of exciting actions of each player is usually very large (equal to J^s; if s is the number of activities belong to path and J is number of duration distribution related to each activity), the use of methods based on the complete set of actions, will lead to very long computational times. Thus, we presented a simple algorithm with additional restrictions to tackle the computational complexity.

4.1. The algorithm

The algorithm starts with an initial and feasible solution which feasible actions for all players are selected arbitrarily. Next, in an iterative process, the best response of each player determines the last action of other players. Therefore, if in the initial actions, the action belongs to player n is worst action; the algorithm determines best response of player n provided that the competitor's strategy are not changed. Then in next iteration, it computes player n' best response to other player actions which the player n' has a worst action and so on. The algorithm continues in this way until it reaches a situation in which each player's action is the best response to the other ones.

1. Determine initial arbitrary feasible actions for all players.
2. Let $k \leftarrow 1$, and let $r_{1,n}$ be an arbitrary feasible action for player n and $w_{1,n}$ is the payoff (reliability) related to $r_{1,n}$. This payoff ($w_{1,n}$) is a worst payoff than the other player's payoff ($w_{1,n} \leq w_{1,n'} | n' = 1, \dots, N$).
3. Find best response of player n to $r_{1,n'}$ ($n' \neq n$).
4. Find the player n with worst payoff ($w_{k,n} \leq w_{k,n'} | n' = 1, \dots, N$)
5. $k \leftarrow k + 1$
6. Compute best response of player n ($r_{k,n}$) to $r_{k-1,n'}$ ($n' \neq n$) with the following restriction:
   $w_{k,n'} \geq w_{k-1,n} + \Delta$
7. If $w_{k,n} = w_{k-1,n}$ and $\Delta \geq \varepsilon$ (ε is a small number) then $\Delta \leftarrow {}^{\Delta}/_{2}$ and go to 6.
8. If $w_{k,n} = w_{k-1,n}$ and $\Delta < \varepsilon$ then ($w_{k,n'} | n' = 1, \dots, N$) is a Nash equilibrium for the simultaneous decision of players. Stop.
9. Go to 4.

4.2. Computing the best responses

In order to be able to use the algorithm, we must have a way to compute the best response of a player to the other one's strategy. This can be done by solving a mixed integer programming problem. In each iteration the strategy of player n with worst payoff is corrected so that maximizes its payoff value with keeping the strategy of other players. Therefore, the $\max(1 - p(A_n))$ is replaced with relation (10) and the following restrictions are added to model 2. Constraint (12) ensures that the strategies of competitors are fully preserved. In other words, we assume in iteration k of algorithm, each action for player n' that maintains its payoff greater than $w_{k-1,n} + \Delta$ is belong to unit strategy. Therefore, in iteration *k,* each player n' has two types of selectable strategies including the sets of $\{y_{j,s} | s \in$

$E_{n'}$ & $j = 1, \dots, j_s\}$ so that $\left(1 - p(A_{n'})\right) \geq w_{k-1,n} + \Delta$ and the sets of $\{y_{j,s} | s \in E_{n'}$ & $j = 1, \dots, j_s\}$ so that $\left(1 - p(A_{n'})\right) < w_{k-1,n} + \Delta$. In this relation Δ is a positive number that is determined experimentally. This parameter assists to prevent a cycle and reduce the time to reach equilibrium.

$$\left(1 - p(A_{n'})\right) \geq w_{k-1,n} + \Delta \qquad n' = 1, .., N \ \& \ n' \neq n \tag{12}$$

The following restriction as a cutting plane assists to reduce the time for reach equilibrium.

$$\sum_{j=j_{k-1,s}+1}^{j_s} y_{j,s} \leq 0 \qquad n' = 1, .., N \ \& \ n' \neq n \ ; \ s \notin T_{n,n'} \& s \in E_{n'} \tag{13}$$

Which the $j_{k-1,s}$ is the duration distribution mode that assigned to activity s in iteration k-1 of algorithm.

5. Numerical experiment

In this section the performance and effectiveness of the proposed algorithm and the credibility and performance of the proposed mathematical models (Model 1 & 2) are evaluated and achieved Nash-equilibrium points compared with optimal solutions. The mathematical models and proposed algorithm are coded in the GAMS 24.1.2 software and solved by Bonmin solver on a PC with 2.66 GHz processor and 4 G of RAM. Furthermore, to demonstrate the effectiveness of proposed solution methods the Relative Percentage Deviation (RPD) criterion is used. A random generation procedure was used to generate the problem instances. In this procedure after determining the number of project activities, each activity can be a predecessor for one to three activities that are selected randomly. Also, each activity can be a successor for one to three activities that are selected randomly. In this process, care is needed to avoid creating a loop of activities. Table 1 illustrates the way that other parameters of these instances are produced.

Table1. Simulation parameters of problem instances.

parameter	produced by
Duration of activities in basic scheduling (B_S)	discrete uniform distribution between 5 to 100
Minimum and Maximum duration of activities	$\left[B_S - \text{rand()}\, {}^{B_S}/_2 , B_S + \text{rand()}\, {}^{B_S}/_2\right]$
Parameters α and β for Beta distributions	Uniform distribution between 2 to 8
Redundant resource needed to perform activity s with time distribution j	Uniform distribution between 5 to 15
Capacity of redundant resource	$S \times$ Uniform distribution between 5 to 10

To compare the results of original model (model 1) and replaced model (model 2), we present the results of several numerical example. The obtained results show the accuracy of the model 2 and proposed algorithm. Figure 3 shows the reliability of a project with two paths considering the resource capacity. In this figure, the horizontal axes show the probability of delay for paths 1 and 2. Also vertical axis shows the reliability of project. The optimal reliability of project for this instance is equal to 0.879 while the value obtained from model 2 and proposed algorithm (Nash Equilibrium) is equal to 0.877. So, for this example the RPD is about 0.2%. The more results are presented in Table 2 and 3. According to the results presented in tables 2 and 3, we compared the obtained results from proposed algorithm with result of model 2 as a viable alternative for original model. Therefore, we generate 9 experimental instances in order to evaluate the performance of the proposed algorithm, and also assessing the efficiency of the mathematical model 2. The results of model 2 and the proposed algorithm are shown in Table 4 to 6 which they are classified in three sizes (small, medium and large). Both the solution quality and the efficiency of the proposed procedures depend on the size of these parameters. In order to determine the best trade-off between algorithms' speed and solution quality the runtime limit of 1800 seconds is imposed on solution methods.

The comparison the results of the proposed algorithm with the results obtained by model 2, shows that the proposed algorithm has better performance in the computational time and the solution quality. Therefore, proposed algorithm is an efficient algorithm to overcome the model 2 computational complexity. However, the average time for the proposed algorithm in small, medium and large size instances are 108.3, 965.3 and >1800 seconds respectively. These times in model2 are 179.3, >1311 and >1800 seconds. Also, the RPD for solution quality in proposed algorithm is on average 0.44% better than model 2 in medium size instances. This value is 9.3% for large size instances. Also, it can be concluded that the restrictions (12) and (13) make a positive effect for reducing computational time by cutting a substantial part of the solution space.

Table 2. Compare the results of model 1 and 2.

Instance *S-N-J* *	Reliability Model 1	Reliability Model 2	RPD%
10:2:30	0.921	0.921	0.00%
15:2:60	0.879	0.877	0.23%
20:3:80	0.835	0.831	0.48%
30:3:100	0.848	0.844	0.47%

* *S-N-J* : *S* the total number of project activities – *N* the total number of project paths - *J* the total number of duration distribution mode

Table 3. Compare the CPU time of model 1 and 2.

Instance *S-N-J*	Time (s) Model 1	Time (s) Model 2	RPD%
10:2:30	18	12	50%
15:2:60	342	54	533%
20:3:80	741	112	562%
30:3:100	1037	272	281%

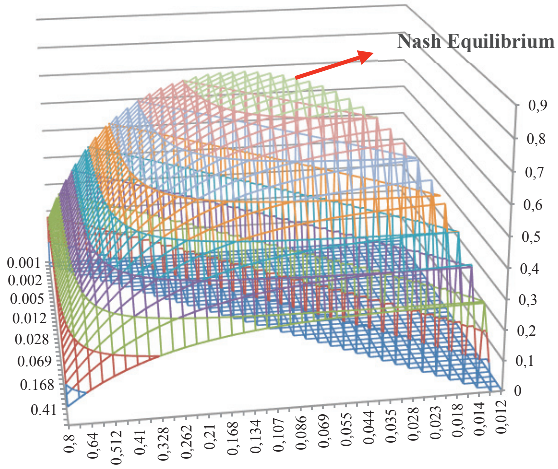

Fig. 3. The optimum reliability value for a sample project with two path.

Table 4. Compare the obtained result by model 2 and proposed algorithm in small size instances.

Instance *S-N-J*	Reliability			Time (s)		
	Model 2	Proposed algorithm	RPD%	Model 2	Proposed algorithm	RPD%
15:2:60	0.877	0.877	0.00%	54	43	25.6%
20:3:80	0.831	0.831	0.00%	112	85	31.8%
30:3:100	0.844	0.844	0.00%	372	197	88.8%

Table 5. Compare the obtained result by model 2 and proposed algorithm in medium size instances.

Instance *S-N-J*	Reliability			Time (s)		
	Model 2	Proposed algorithm	RPD%	Model 2	Proposed algorithm	RPD%
40:3:150	0.892	0.892	0.00%	792	427	85.5%
50:4:180	0.873	0.873	0.00%	1341	943	42.2%
60:5:220	0.824	0.835	1.32%	>1800	1526	>18.0%

Table 6. Compare the obtained result by model 2 and proposed algorithm in large size instances.

Instance *S-N-J*	Reliability			Time (s)		
	Model 2	Proposed algorithm	RPD%	Model 2	Proposed algorithm	RPD%
80:8:300	0.721	0.816	11.6%	>1800	>1800	0.00%
90:9:350	0.685	0.755	9.3%	>1800	>1800	0.00%
100:10:400	0.692	0.746	7.2%	>1800	>1800	0.00%

6. Conclusion

In this paper, we studied the reliable project scheduling problem under uncertainty and developed competitive conditions based game-theoretical approach which all of players (project path) are competed with each other on attracting the more resource. A new mathematical model is provided and due to the computational complexity of problem an algorithmic procedure is proposed which has efficient performance in achieving the Nash-equilibrium points. Finally, the computational results show that the proposed algorithm has satisfactory performance in two criteria including computational time and the quality of the final results. Development of the new exact methods to solve the problem is a suitable contribution for future research.

Reference:

[1] Vahedi-Nouri, Behdin, Parviz Fattahi, Mohammad Rohaninejad, and Reza Tavakkoli-Moghaddam. "Minimizing the total completion time on a single machine with the learning effect and multiple availability constraints." *Applied Mathematical Modelling* 37, no. 5 (2013): 3126-3137.

[2] Rohaninejad, Mohammad, Amir Saman Kheirkhah, Behdin Vahedi Nouri, and Parviz Fattahi. "Two hybrid tabu search–firefly algorithms for the capacitated job shop scheduling problem with sequence-dependent setup cost."*International Journal of Computer Integrated Manufacturing* 28, no. 5 (2015): 470-487.

[3] Rohaninejad, Mohamad, Amirsaman Kheirkhah, Parviz Fattahi, and Behdin Vahedi-Nouri. "A hybrid multi-objective genetic algorithm based on the ELECTRE method for a capacitated flexible job shop scheduling problem."*The International Journal of Advanced Manufacturing Technology* 77, no. 1-4 (2015): 51-66.

[4] Rohaninejad, Mohammad, Amirsaman Kheirkhah, and Parviz Fattahi. "Simultaneous lot-sizing and scheduling in flexible job shop problems." *The International Journal of Advanced Manufacturing Technology* 78, no. 1-4 (2015): 1-18.

[5] Herroelen, Willy, and Roel Leus. "Project scheduling under uncertainty: Survey and research potentials." *European journal of operational research* 165, no. 2 (2005): 289-306.

[6] Alagöz, Oğuzhan, and Meral Azizoğlu. "Rescheduling of identical parallel machines under machine eligibility constraints." *European Journal of Operational Research* 149, no. 3 (2003): 523-532.

[7] Fang, Chen, Rainer Kolisch, Ling Wang, and Chundi Mu. "An estimation of distribution algorithm and new computational results for the stochastic resource-constrained project scheduling problem." *Flexible Services and Manufacturing Journal* (2012): 1-21.

[8] Wang, Juite. "A fuzzy robust scheduling approach for product development projects." *European Journal of Operational Research* 152, no. 1 (2004): 180-194.

[9] Lamas, Patricio, and Erik Demeulemeester. "A purely proactive scheduling procedure for the resource-constrained project scheduling problem with stochastic activity durations." *Journal of Scheduling* (2014): 1-20.

[10] Penz, Bernard, Christophe Rapine, and Denis Trystram. "Sensitivity analysis of scheduling algorithms." *European Journal of Operational Research* 134, no. 3 (2001): 606-615.

[11] Brčić, Mario, Damir Kalpić, and Krešimir Fertalj. "Resource constrained project scheduling under uncertainty: a survey." In *23rd Central European Conference on Information and Intelligent Systems*. 2012.

[12] Chaari, Tarak, Sondes Chaabane, Nassima Aissani, and Damien Trentesaux. "Scheduling under uncertainty: Survey and research directions." In *Advanced Logistics and Transport (ICALT), 2014 International Conference on*, pp. 229-234. IEEE, 2014.

[13] Lin, Xiaoxia, Stacy L. Janak, and Christodoulos A. Floudas. "A new robust optimization approach for scheduling under uncertainty:: I. Bounded uncertainty." *Computers & chemical engineering* 28, no. 6 (2004): 1069-1085.

[14] Lambrechts, Olivier, Erik Demeulemeester, and Willy Herroelen. "Proactive and reactive strategies for resource-constrained project scheduling with uncertain resource availabilities." *Journal of scheduling* 11, no. 2 (2008): 121-136.

[15] Lambrechts, Olivier, Erik Demeulemeester, and Willy Herroelen. "A tabu search procedure for developing robust predictive project schedules." *International Journal of Production Economics* 111, no. 2 (2008): 493-508.

[16] Lambrechts, Olivier, Erik Demeulemeester, and Willy Herroelen. "Time slack-based techniques for robust project scheduling subject to resource uncertainty." *Annals of Operations Research* 186, no. 1 (2011): 443-464.

[17] Ji, Xiaoyu, and Kai Yao. "Uncertain project scheduling problem with resource constraints." *Journal of Intelligent Manufacturing* (2014): 1-6.

[18] Fu, Na, Hoong Chuin Lau, and Pradeep Varakantham. "Robust execution strategies for project scheduling with unreliable resources and stochastic durations." *Journal of Scheduling* (2015): 1-16.

[19] Kallenberg, Olav. *Foundations of modern probability*. Springer Science & Business Media, 2006.